CBCS SCHEME

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20SCN/SCE/SSE/SCS/SIT/SIS/SFC/LNI/SAM/SCR11

First Semester M.Tech. Degree Examination, Jan./Feb. 2021
Mathematical Foundations of Computer Science

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define linearly independent and linearly dependent vectors.
Show that the vectors $v_1 = (1, 1, 2, 4)$, $v_2 = (2, -1, -5, 2)$, $v_3 = (1, -1, -4, 0)$ and $v_4 = (2, 1, 1, 6)$ are linearly dependent in $R^4(R)$. **(06 Marks)**

b. Define the terms basis and dimension. Find the basis for the subspace W spanned by $\{v_1, v_2, v_3, v_4\}$.

$$v_1 = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix},\ v_2 = \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix},\ v_3 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix},\ v_4 = \begin{bmatrix} -4 \\ -8 \\ 9 \end{bmatrix}$$

Also find dimension of W. **(07 Marks)**

c. Define linear transformation. Linear transformation defined by $T: R^2 \to R^3$ by $T(x) = Ax$,

$$\text{Let } A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix},\ u = \begin{bmatrix} 2 \\ -1 \end{bmatrix},\ b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$$

Find T(u), the image of u under the transformation T. Also find an x in $\mathbb{R}^2$ whose image under T is b. **(07 Marks)**

OR

2 a. Define vector space and subspace. **(06 Marks)**

b. Define the term coordinate system. Find the coordinate vector $[x]_B$ of x relative to the given basis

$$b_1 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix},\ b_2 = \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix},\ b_3 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix},\ x = \begin{bmatrix} 8 \\ -9 \\ 6 \end{bmatrix}$$ **(07 Marks)**

c. The set $B = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for P_2. Find the coordinate vector of $P(t) = 1 + 4t + 7t^2$ relative to B. Also test the linear independence of the set of polynomials. **(07 Marks)**

Module-2

3 a. Verify that $\{u_1, u_2\}$ is an orthogonal set, and then find the orthogonal projection of y onto span $\{u_1, u_2\}$.

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},\ u_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix},\ y = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$$ **(06 Marks)**

b. Let $u_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix},\ u_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\{u_1, u_2\}$ is an orthogonal basis for $W = \text{span}\{u_1, u_2\}$. Write y as the sum of a vector in W and a vector orthogonal to W. **(07 Marks)**

c. If $A = \begin{bmatrix} 3 & 1 \\ 6 & 2 \\ 0 & 2 \end{bmatrix}$. Find the QR factorization of A. **(07 Marks)**

OR

4 a. Define orthogonal basis. Show that $\{u_1, u_2, u_3\}$ is an orthogonal basis for $\mathbb{R}^3$. Then express x as a linear combination of the vector u_1, u_2, u_3.

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\ u_2 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix},\ u_3 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix},\ x = \begin{bmatrix} 8 \\ -4 \\ -3 \end{bmatrix}$$ **(06 Marks)**

b. Find an orthogonal basis for the column space of the matrix

$$\begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$$ using Gram-Schmidt process. **(07 Marks)**

c. Find a least square solution of $Ax = b$ for $A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}$ **(07 Marks)**

Module-3

5 a. Make a change of variable $x = py$, that transforms the quadratic form $x_1^2 - 8x_1 x_2 - 5x_2^2$ into a quadratic form with no cross product term. **(10 Marks)**

b. Find a singular value decomposition of $A = \begin{bmatrix} 4 & 11 & +14 \\ 8 & 7 & -2 \end{bmatrix}$ with eigen values of $A^T A$ are 360, 90 and 0. **(10 Marks)**

OR

6 a. Find:

i) Maximum value of Q(x) subject to the constraint $x^T x = 1$

ii) A unit vector u where this maximum is attained

iii) The maximum of Q(x) subject to the constraints $x^T x = 1$ and $x^T u = 0$. Given $Q(x) = x_1^2 + x_2^2 - 10x_1 x_2$ **(10 Marks)**

b. Three measurements are made on each of four individuals in a random sample from a population. The observation vectors are $X_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $X_2 = \begin{bmatrix} 4 \\ 2 \\ 13 \end{bmatrix}$, $X_3 = \begin{bmatrix} 7 \\ 8 \\ 1 \end{bmatrix}$, $X_4 = \begin{bmatrix} 8 \\ 4 \\ 5 \end{bmatrix}$.

Compute the sample mean and the covariance matrix. **(10 Marks)**

Module-4

7 a. Compute the coefficient of correlation between X and Y using the following data:

X:	1	3	5	7	8	10
Y:	8	12	15	17	18	20

(06 Marks)

b. A study of prices of rice at Chennai and Mumbai gave the following data:

	Chennai	Mumbai
Mean	19.5	17.75
SD	1.75	2.5

Also the coefficient of correlation between the two is 0.8. Estimate the most likely price of rice (i) at Chennai corresponding to the price of 18 at Mumbai (ii) at Mumbai corresponding to the price of 17 at Chennai. **(07 Marks)**

c. Fit a curve of the form $y = ax^b$ to the following data and estimate y at x = 12.

x :	20	16	10	11	14
y :	22	41	120	89	56

(07 Marks)

OR

8 a. Find the angle between the two lines of regression. **(06 Marks)**

b. Fit an equation of the form $y = a_0 + a_1x_1 + a_2x_2$ to the given data:

x_1	1	2	3	4
x_2	10	1	2	3
y	12	18	24	30

(07 Marks)

c. X_1, X_2, X_3 are three variates measured from their means with N = 10, $\sum X_1^2 = 90$, $\sum X_2^2 = 160$, $\sum X_3^2 = 40$, $\sum X_1X_2 = 60$, $\sum X_2X_3 = 60$, $\sum X_1X_3 = 40$. Calculate the multiple correlation coefficient $R_{1.23}$ **(07 Marks)**

Module-5

9 a. If the random variable X takes the values 1, 2, 3 and 4 such that 2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4). Find the probability distribution and $P(1 \le X < 3)$ and $P(X \ge 3)$ **(06 Marks)**

b. A continuous random variable X has a p.d.f $f(x) = Kx^2e^{-x}$, $x \ge 0$. Find K, mean and variance. **(07 Marks)**

c. Fit a binomial distribution for the following data and also test the goodness of fit.

x :	0	1	2	3	4	5	6	Total
f :	5	18	28	12	7	6	4	80

To find the binomial distribution $N(p + q)^n$, which fits the given data, $\left(\chi^2_{0.02}\ (\nu = 2) = 5.99\right)$. **(07 Marks)**

OR

10 a. The following data represents the biological values of protein from cow's milk and buffalo's milk at a certain level.

Cow's milk	1.82	2.02	1.88	1.61	1.81	1.54
Buffalo's milk	2.00	1.83	1.86	2.03	2.19	1.88

Examine if the average values of protein in the two samples significantly differ $(t_{0.05}\ (\nu = 10) = 2.23)$ **(06 Marks)**

b. The following data give the number of air craft accidents that occurred during the various days of a week:

Days:	Mon	Tues	Wed	Thu	Fri	Sat
No. of accidents:	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week $\left(\chi^2_{0.05}(\nu = 5) = 11.07\right)$. **(07 Marks)**

c. The probability density function of a variable X is

X :	0	1	2	3	4	5	6
P(X) :	k	3k	5k	7k	9k	11k	13k

Find k and $P(X < 4)$, $P(X \ge 5)$, $P(3 < X \le 6)$ **(07 Marks)**

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